Northampton Academy Trust Mathematics Aims and Principles
February 2020

Mathematics at NPAT is based on the Trust's vision to achieve educational excellence. It is the Trust's aspiration for all children to achieve highly, for every school to be a great school and every individual to continually improve. Daily maths sessions form an essential part of the trust's mission to develop a multi-academy trust of highly effective and sustainable schools.

Mastering maths means pupils acquiring a deep, long-term, secure and adaptable understanding of the subject. Within the mathematics curriculum sits a firm foundation of equality. With the right support and a culture of high expectations, a vast majority of children are able to achieve the age-related expected standard (Unless for specific reasons e.g. SEND). Embracing this belief alongside effective quality first teaching, tends to result in greater success.

At NPAT, we aspire for all learners to become confident mathematicians who are critical thinkers and problem solvers, not only whilst in their primary education but in their secondary school and beyond.

This document has been compiled alongside research completed by The Education Endowment Foundation and The National Centre for Excellence in the Teaching of Mathematics.

Principles for Effective Teaching

Children's chances of success are maximised if they develop deep and lasting understanding of mathematical procedures and concepts. To achieve this, this document aims to provide clarity of the trust's shared vision and pedagogy. It is the Trust Maths Lead, Head teachers and School Maths Leads responsibility to share this vision, ensure training is up to date within individual schools and to monitor the effectiveness of teaching and learning.

To ensure clarity for all staff, long term plans within each school maps out the coverage and progression for each year group in line with the National Curriculum. From the long-term progression document, it is good practice to break down concepts into smaller steps to gain a deep understanding of each building block required to understand the wider concept. Effective practice will ensure the relationship between the building blocks are explicitly modelled and shared with the children so they see maths as a continuum rather than seeing new learning as stand-alone procedures.

As a trust we have identified key elements for effective practice that weave together within lessons:

Key Elements for Effective Teaching					
Vocabulary and Discussion	**Challenge for All**	**Use of Concrete, Pictorial and Abstract Representations**	**Variation**	**Reasoning and Problem Solving**	**Fluency and Arithmetic**

Each element has been explained to share the pedagogy behind them and to see some examples of how this looks when teaching a concept, see Appendix 1.

Vocabulary and Discussion

To support a child's understanding of mathematics, precise mathematical vocabulary needs to be modelled, explored and defined, to enable expectation that children will use this within their own explanations and reasoning. Accurate use of vocabulary supports the children's ability to work through unfamiliar problems and look critically at which steps they could take to solve it.

Within lessons, opportunities for high quality collaboration amongst peers can support exploration of key concepts to build up to a strong understanding of efficient methods. Lessons carefully designed to engage learners and ensure all are active members within the lesson throughout, supports the teacher to build frequent opportunities for Assessment for Learning (AfL) and can make certain that all children are taken on the same conceptual journey to reach at least the expected standard.

Stem Sentences

Modelling and encouraging verbal 'stem sentences' in lessons helps to build independence by offering a scaffold a child can use when reasoning about a concept. Teachers need to ensure they do not over generalise a concept and cause misconceptions later on e.g. 'when we multiply two numbers, the product is larger' as when multiplying fractions this will not be accurate.
Stem sentences can be used:

- **To secure understanding:** By answering a question in a full sentence e.g. 'What is the difference between 7 and 12? The difference between 7 and 12 is….' Or 'What does the 3 represent in my question? The 3 represents ….' This should be used in every lesson to secure the reasoning skills as they answer.
- **To scaffold learning:** A rule to remember e.g. 'To find 10% of a whole we divide by 10', '360 degrees is the same as a full turn', 'A fraction is an equal part of a whole.'
- **Similar to a writing frame**: To apply to the questions they are doing e.g. 'In the number ____ the ____ digit represents _____ tens' or ' _____ multiplied by 2 is the same
- as ____ doubled.'

Challenge for All

Challenge for all is an important feature within a lesson. Effective teachers design lessons, showing how they can scaffold each small step of learning within the lesson for children requiring extra support. Assessment opportunities and key questions should identify these children quickly and misconceptions / mistakes addressed within the lesson where possible. If this is not possible, it is more effective to plan intervention opportunities before the next lesson to ensure the gap does not widen further.

When a child is showing a sound understanding for a concept, further challenge through questioning or a task is required to extend their thinking further.

Questions could include:

- 'What if…?'
- 'Can you show me using a different representation?'
- 'Prove it!'
- Missing number challenges.
- Being exposed to a new unfamiliar context to apply the concept to.

All challenge should deepen an understanding for the concept and children should not be accessing another year group's curriculum. If a child has understood a concept deeply, it is also not challenging for a child to complete the same process with larger numbers.

Within classrooms, working walls are a good way to scaffold and build on learning in lessons. Displaying key vocabulary, relevant structures (e.g. pictorial representations) and efficient methods can allow children to refer back and build independence and problem solving skills when applying new learning to unfamiliar contexts.

Use of Concrete, Pictorial and Abstract Representations *(CPA)*

"The aim is to use manipulatives and representations to reveal mathematical structures and enable pupils to understand and use maths independently." EEF, Improving mathematics in Keys Stages Two and Three.

To support access for all and equality within the classroom, NPAT advocate the use of concrete, pictorial and abstract representations with the children to deepen their understanding, showing them the 'how' and 'why' and not just the 'what'. The theory of 'The answer is just the beginning' should support the teacher to unpick concepts rather than just accepting 'correct answers'.

Thoroughly exposing key concepts within each lesson through this approach supports children's conceptual understanding. This in turn should allow children to apply knowledge to varying contexts and draw on strategies / procedures more flexibly.

Concrete, pictorial and abstract representations should be effectively woven together to show the relationship between them. Key questions, e.g. 'what is the same and what is different?', 'What do you notice?' allow teachers to explicitly identify and model the key essence of a concept.

Concrete Representations

It is an important stage to every child's conceptual development that they all have opportunities to physically manipulate resources to understand what is happening within a process. Concrete resources should be used within all year groups to support all learners and available throughout each unit.

Teachers should:

- Choose manipulatives purposefully to ensure they enhance the children's conceptual understanding. Incorrect manipulative choice can cause misconceptions and / or a cognitive overload for the learners.
- Select manipulatives that are specifically chosen to address misconceptions.
- Ensure access to the equipment for all learners at all stages of the lesson and not just chosen for certain groups.
- Model positive attitudes towards concrete manipulatives so not seen as a weakness by the learners.
- Use a range of representations appropriately.

Pictorial Representations

Pictorial Representations allow learners to represent their concrete manipulation or their understanding of a question in an image. This may be an informal image / mark making at first to start to unpick a concept. Children who are able to see the relationships between the concrete and the pictorial structures will be more likely to be able to understand what is happening when it is represented in abstract form.

Teachers should:

- Model the relationship between the concrete and the pictorial representations.
- Model efficient representations that supports the child's understanding e.g. part/ part whole, bar models, place value charts.
- Encourage the children to record pictorial representations to show their working.

Through AfL opportunities teachers are able to see which children are able to work flexibly between the two and which children require further support at a concrete level.

Abstract Representations

The aim is that children are able to function in an abstract manner to problem solve, reason and apply their knowledge confidently. When working in the abstract the children should be able to understand what the process is and be able to reason to explain their workings confidently, so it is vital the previous stages are valued and given the time they need.

Teachers should:

- Model the relationship between the abstract representation alongside the concrete and the pictorial structures.
- Understand that the movement through the different stages is not hierarchical.
- Refer to the school calculation policy to use appropriate formal methods to their year group.

Recording in Mathematics

It is the school's decision whether children use plain paper books or squared paper books to support any recording. Teachers need to consider what will support the learners and be flexible with this to support the children's needs e.g. allowing children to use squared paper if it will support their accuracy with place value. On the other hand, squared paper should not restrict a child's creativity when problem solving or creating pictorial representations. Books should represent the child's independent journey of the lesson and may include any pictorial or abstract representations that have helped them understand a concept.

Variation

The building blocks with a lesson need to be designed carefully to ensure appropriate progression and a deep understanding at each stage. Teachers who plan for variation in their lessons ensure contexts are varied widely which in turn builds more resilience when approaching unknown problem solving situations. By explicitly drawing out key elements and relationships of a concept allows for independent generalisations, which supports the retention of the information.

The two strands of variation are:

- Conceptual variation
- Procedural variation

Conceptual Variation

Definition: Understanding concepts from multiple perspectives.

Conceptual variation links closely to the CPA approach and looks at different ways concepts can be represented. Not only is it important to observe and process 'what a concept is', it is also just as important to know what 'it isn't' so that children focus on the key essence and do not build assumptions, e.g. when looking at fractions, show parts equally split and unequally to discuss and embed the need for equality in the parts.

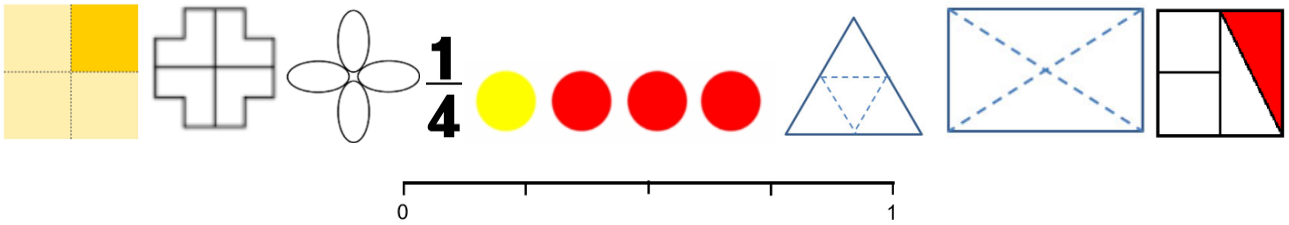
Planning for misconceptions (Non-conceptual variation) ensures over generalisations are not made, it helps children understand exactly what the concept is and allows them to work flexibly and with a strong understanding when new, unknown situations arise.

Example Conceptual Variation of a Fraction:

Exploring the concept of one quarter: What different ways can ¼ be represented? Children require time to explore in depth and challenge their thinking e.g. can it still be a quarter if the parts look different? How can they prove this using concrete and pictorial representations?

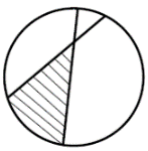
Stem sentences can be used to scaffold their reasoning skills from their exploration.

"The whole has been divided into 4 equal parts. Each of those parts is one quarter of the whole."

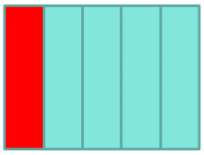


Non-conceptual Variation:

Provides opportunities for misconceptions to be explored. At a first glance it may look like a quarter, but it is not. Can the same stem sentence be applied? If not, why not? This builds a child's chance to explore more deeply and challenge ideas. Reasoning skills can be applied when explaining whether it is conceptual or non-conceptual.



"The whole has been divided into 4 parts but they are not equal."



"The whole has been divided into 5 equal parts. Each part if one fifth of the whole."

Note: For more examples please refer to the appendices

Procedural Variation:

Definition: The small steps taken to build up to the understanding of a concept.
"The smaller the distance from the existing knowledge and the new learning, the greater the success", Gu, L. (1994).

When planning, good lessons will take into consideration the outcome required at the end of the lesson or sequence of lessons. To achieve this, effective teachers will ensure an appropriate journey to build up to this final outcome securely. Small conceptual steps are taken to explicitly share relationships between previous learning and how this now links to the new learning.

Within procedural variation it can be useful to 'fix' a number or a representation to look at it from different perspectives, e.g. 'I know ____ so I can use it to solve _____'. When learning a new concept, teachers should consider the numbers that they use to expose the process. This should incorporate numbers that children will find manageable and accessible before applying this to numbers suitable for age related expectations. This ensures the process/concept is explicit and to also lessen the cognitive load of dealing with numbers that may not be accessible for all straight away.

Example Procedural Variation for Fractions:
Small steps taken to start to understand fractions fully could include:

1. Learning about parts of a whole (shapes and numbers)
2. Understanding the part has to be equal to be a fraction
3. Making equal parts
4. Identifying equal parts
5. Learning the names of a unit fraction e.g. one half
6. Learning the fraction symbol for unit fractions e.g. ½

Note: For more specific concept examples please refer to the appendices

Question Design

Procedural variation is also linked to question design for independent application tasks. Intelligent practice is where teachers design questions to work through the concepts building on relationships exposed in the quality first teaching.

Independent tasks:

- Should refer directly to the concept in the lesson.
- Should have a variety of questions so they are not repeating the same process unnecessarily.
- Should have an opportunity to reason and /or problem solve.
- Should have an opportunity to apply their new knowledge in a different context.
- May have an opportunity to apply fluency skills.

"In designing these exercises avoid mechanical repetition, but instead practice the thinking process with increasing creativity." Gu (2004)
Every child must have the opportunity to achieve within the independent activities so that no ceilings are put on a child's learning and to also supports equality. Concrete and pictorial representations are still acceptable to use in independent application if a child requires this.

Variation in Strategy Sharing

Lastly procedural variation is associated with the various strategies that are used within the class to solve problems and questions. Sharing a variety of strategies, again supports equality and using the theory '*the answer is only the beginning*', children begin to see that maths is a creative subject. Class discussions illustrate that there are many solutions and strategies to find accurate responses. It is the teacher's role to; facilitate the strategies being shared, model effective approaches to show the structure of the mathematics and then demonstrate efficient methods.

Reasoning and Problem Solving

"Problem solving generally refers to situations in which pupils do not have a readily available method that they can use. Instead they have to approach the problem flexibly and work out a solution for themselves." EEF, Improving mathematics in Keys Stages Two and Three.

All learners should have the opportunity to reason and problem solve in every lesson. To understand a concept fully, children need to be able to flexibly apply learnt concepts to new and unfamiliar situations. If this is not possible, the concept has not been understood at an appropriate level and further support / scaffolding will be required within the small steps before moving to a problem solving situation.

Reasoning requires a child to explain their thinking around a question. Opportunities could include 'prove it', 'true or false' or 'explain it' opportunities to build reasoning skills. As stated previously, accurate use of vocabulary should be encouraged and, if required, scaffolding of stem sentences can be used to structure the children's ability to explain their thinking accurately. Effective practice will ensure opportunities for reasoning for all learners.

Reasoning opportunities can vary from answering a question by explaining a range of answers: the answer itself, why a method was chosen or an explanation around a misconception e.g. 'I think the answer is wrong because…'. It should not be one question at the end of the lesson that only the higher attainers work on.

Reasoning and problem solving within lessons:

- Should be for all learners not just children working above age related expectation.
- Should be used to deepen the understanding of concepts by applying a child's knowledge to varied contexts.
- Should expose possible misconceptions to ensure a deep understanding.
- Should expose the children to problems with real life situations where possible to show the relevance and importance of mathematics in their own lives.
- Is not solely linked to word problems - a problem is defined by having to apply logic and a process e.g. finding a missing number, finding different solutions, being exposed to a new context or working systematically to find all possible solutions.

When exploring word problems, assumptions regarding vocabulary (mathematical and contextual) should not be made and appropriate time should be given to exploring any given word problems. This weaves the importance of vocabulary, collaboration and making time to share effective strategies with the whole class.

Fluency and Arithmetic

Fluency encompasses efficiency, accuracy and flexibility and requires more than memorisation of a single fact or procedure. Being fluent in maths allows children to identify relationships within their learning and apply known facts to support finding solutions. Quick and efficient recall of facts and procedures is important in order for learners to keep track of problems, think strategically and solve problems.

Fluency also demands the flexibility to move between different contexts and representations of mathematics, to recognise relationships and make connections and to make appropriate choices from a whole toolkit of methods, strategies and approaches.

To support fluency, as a trust we should provide opportunities to recap on concepts taught through arithmetic sessions. This enables children to practice previously learnt skills including practising number bonds, multiplication facts or formal written methods and support embedding them into their long term memory. This could be linked to the maths lesson for that day as an introduction or plenary; or beyond the maths lesson to cover any maths areas covered so far. Being able to apply known facts to a problem supports the child's cognitive load thus enabling them to focus on the conceptual understanding rather than having to understand new concepts alongside having to complete calculations.

"Quick retrieval of number facts is important for success in mathematics. It is likely that pupils who have problems retrieving addition, subtraction, multiplication and division facts, including numbers bonds and multiples, will have difficulty understanding and using mathematical concepts they encounter later on in their studies." EEF, Improving mathematics in Keys Stages Two and Three.

School Documentation and Support

Each school should have a School Strategy for Maths. The aim of the strategy is to inform all stakeholders within each specific school of how they reflect the 'Maths Aims and Principles' within their setting. Schools are to make the decision with the planning support and/or schemes that they implement and should ensure monitoring is completed to review the effectiveness of this.

Included as part of this documentation, there needs to be:

- **The Maths Strategy**: to outline the intent for mathematics and all non-negotiable expectations within a school context e.g. timetable expectations, choice of planning support, to show how the school will meet each key element within this document, how key learning behaviours will be encouraged to support their progress in primary school and beyond with maths.
- **Maths Action Plan**: to highlight key priorities to implement for the academic year. The Maths Lead and SLT are to ensure continual professional development for teachers to continue to strive to improve the teaching and learning of mathematics to support in the raising on standards.
- **Calculation Policy**: a progression document to outline key calculation strategies for each operation.
- **Long term maps**: to ensure full coverage of the National Curriculum and to show the progression children will follow to build on each concept.
- **Lesson plans**: appropriate to each individual school to ensure teachers are clear on the key concepts taught and to map out the small steps. Schools are able to select a format suitable for their needs and are able to annotate planning available from a scheme if purchased / used as a basis.

All subject leads should have a whole school data picture of mathematics and complete any data analysis using the trust assessment program 'Insight'. This needs to be completed within the year in conjunction with the NPAT Assessment Timeline. Findings from any assessment will need discussion with the Headteacher and SLT, and Trust Maths Lead, with key priorities highlighted for future actions. The Trust Maths Lead will support with this process in maths meetings where appropriate.

Maths Leads will be supported through termly meetings conducted by the Trust Maths Lead and through bespoke support as appropriate.